Investigation 2

Exploring Linear Functions With Graphs and Tables

In the last investigation, you examined relationships that were linear functions. For example, the *distance* a person walks at a constant rate is a function of the amount of *time* a person walks. The *amount of money* a person collects from a walkathon sponsor who pays a fixed amount per *kilometer* is a function of the distance walked. You used tables, graphs, and equations to answer questions about these relationships.

In this investigation, you will continue to solve problems involving linear functions.

2.1 Walking to Win

In Ms. Chang's class, Emile found out that his walking rate is 2.5 meters per second. When he gets home from school, he times his little brother Henri as Henri walks 100 meters. He figured out that Henri's walking rate is 1 meter per second.

Problem 2.1 Finding the Point of Intersection

Henri challenges Emile to a walking race. Because Emile's walking rate is faster, Emile gives Henri a 45-meter head start. Emile knows his brother would enjoy winning the race, but he does not want to make the race so short that it is obvious his brother will win.

A. How long should the race be so that Henri will win in a close race?

B. Describe your strategy for finding your answer to Question A. Give evidence to support your answer.

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2.2 Crossing the Line

Your class may have found some very interesting strategies for solving Problem 2.1, such as:

- Making a table showing time and distance data for both brothers
- Graphing time and distance data for both brothers on the same set of axes
- Writing an equation for each brother representing the relationship between time and distance

How can each of these strategies be used to solve the problem?

What other strategies were used in your class?

Problem 2.2 Using Tables, Graphs, and Equations

A. For each brother in Problem 2.1:

1. Make a table showing the *distance from the starting line* at several different times during the first 40 seconds.

2. Graph the time and the distance from the starting line on the same set of axes.

3. Write an equation representing the relationship. Explain what information each variable and number represents.

B. **1.** How far does Emile walk in 20 seconds?

2. After 20 seconds, how far apart are the brothers? How is this distance represented in the table and on the graph?

3. Is the point (26, 70) on either graph? Explain.

4. When will Emile overtake Henri? Explain.

C. How can you determine which of two lines will be steeper

1. from a table of the data?

2. from an equation?

D. **1.** At what points do Emile's and Henri's graphs cross the y-axis?

2. What information do these points represent in terms of the race?

3. How can these points be found in a table? In an equation?

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Did You Know?

Have you ever seen a walking race? You may have thought the walking style of the racers seemed rather strange. Race walkers must follow two rules:

- The walker must always have one foot in contact with the ground.
- The walker's leg must be straight from the time it strikes the ground until it passes under the body.

A champion race walker can cover a mile in about 6.5 minutes. It takes most people 15 to 20 minutes to walk a mile.

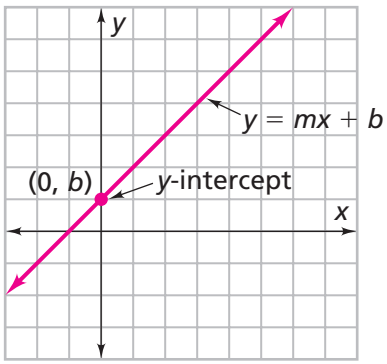
Go Online PHSchool.com **For:** Information about race-walking **Web Code:** ane-9031

2.3 Comparing Costs

In the last problem, you found the point at which Emile's and Henri's graphs cross the y-axis. These points are called the *y-intercepts*.

- The distance d_{Emile} that Emile walks after t seconds can be represented by the equation, $d_{\text{Emile}} = 2.5t$. The y-intercept is $(0, 0)$ and the *coefficient* of t is 2.5.
- The distance d_{Henri} that Henri is from where Emile started can be given by the equation, $d_{\text{Henri}} = 45 + t$, where t is the time in seconds. The y-intercept is $(0, 45)$ and the *coefficient* of t is 1.

All of the linear equations we have studied so far can be written in the form $y = mx + b$ or $y = b + mx$. In this equation, y depends on x.



The **y-intercept** is the point where the line crosses the y-axis, or when $x = 0$. To save time, we sometimes refer to the number b, rather than the coordinates of the point $(0, b)$, as the y-intercept.

A **coefficient** is the number that multiplies a variable in an equation. The m in $y = mx + b$ is the coefficient of x, so mx means m times x.

Problem 2.3 Comparing Equations

Ms. Chang's class decides to give T-shirts to each person who participates in the Walkathon. They receive bids for the cost of the T-shirts from two different companies. Mighty Tee charges \$49 plus \$1 per T-shirt. No-Shrink Tee charges \$4.50 per T-shirt. Ms. Chang writes the following equations to represent the relationship between cost and the number of T-shirts:

$$C_{Mighty} = 49 + n$$
$$C_{No\text{-}Shrink} = 4.5n$$

The number of T-shirts is n. C_{Mighty} is the cost in dollars for Mighty Tee and $C_{No\text{-}Shrink}$ is the cost in dollars for No-Shrink Tee.

A. 1. For each equation, explain what information the y-intercept and the coefficient of n represents.

2. For each company, what is the cost for 20 T-shirts?

3. Lani calculates that the school has about \$120 to spend on T-shirts. From which company will \$120 buy the most T-shirts?

4. a. For what number of T-shirts is the cost of the two companies equal? What is that cost? Explain how you found the answers.

b. How can this information be used to decide which plan to choose?

5. Explain why the relationship between the cost and the number of T-shirts for each company is a linear relationship.

B. The table at the right represents the costs from another company, The Big T.

1. Compare the costs for this company with the costs for the two companies in Question A.
2. Does this plan represent a linear relationship? Explain.
3. **a.** Could the point (20, 84) lie on the graph of this cost plan? Explain.

   b. What information about the number of T-shirts and cost do the coordinates of the point (20, 84) represent?

T-Shirt Costs

n	*C*
0	34
3	41.5
5	46.5
8	54
10	59

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2.4 Connecting Tables, Graphs, and Equations

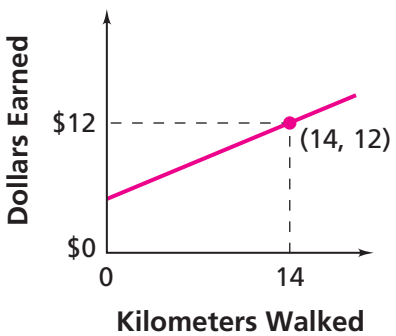
Look again at Alana's pledge plan from Problem 1.3. Suppose A represents the dollars owed and d represents the number of kilometers walked. You can express this plan with the equation below:

$$\text{Alana's pledge plan: } A = 5 + 0.5d$$

Getting Ready for Problem 2.4

- Explain why the point (14, 12) is on the graph of Alana's pledge plan.
- Write a question you could answer by locating this point.
- How can you use the equation for Alana's pledge plan to check the answer to the question you made up?
- How can you use a graph to find the number of kilometers that Alana walks if a sponsor pays her $17? How could you use an equation to answer this question?

Alana's Pledge Plan



In the next problem, you will investigate similar questions relating to pledge plans for a walkathon.

Problem 2.4 Connecting Tables, Graphs, and Equations

Consider the following pledge plans. In each equation, y is the amount pledged in dollars, and x is the number of kilometers walked.

Plan 1 $y = 5x - 3$ Plan 2 $y = -x + 6$ Plan 3 $y = 2$

A. For each pledge plan:

1. What information does the equation give about the pledge plan? Does the plan make sense?

2. Make a table for values of x from -5 to 5.

3. Sketch a graph.

4. Do the y-values increase, decrease, or stay the same as the x-values increase?

B. Explain how you can use a graph, table, or equation to answer Question A, part (4).

C. 1. Which graph from Question A, part (3), can be traced to locate the point (2, 4)?

2. How do the coordinates (2, 4) relate to the equation of the line? To the corresponding table of data?

3. Write a question you could answer by locating this point.

D. 1. Which equation has a graph you can trace to find the value of x that makes $8 = 5x - 3$ a true statement?

2. How does finding the value of x in $8 = 5x - 3$ help you find the coordinates for a point on the line of the equation?

E. The following three points all lie on the graph of the same plan:

$(-7, 13)$ $(1.2, \blacksquare)$ $(\blacksquare, -4)$

1. Two of the points have a missing coordinate. Find the missing coordinate. Explain how you found it.

2. Write a question you could answer by finding the missing coordinate.

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