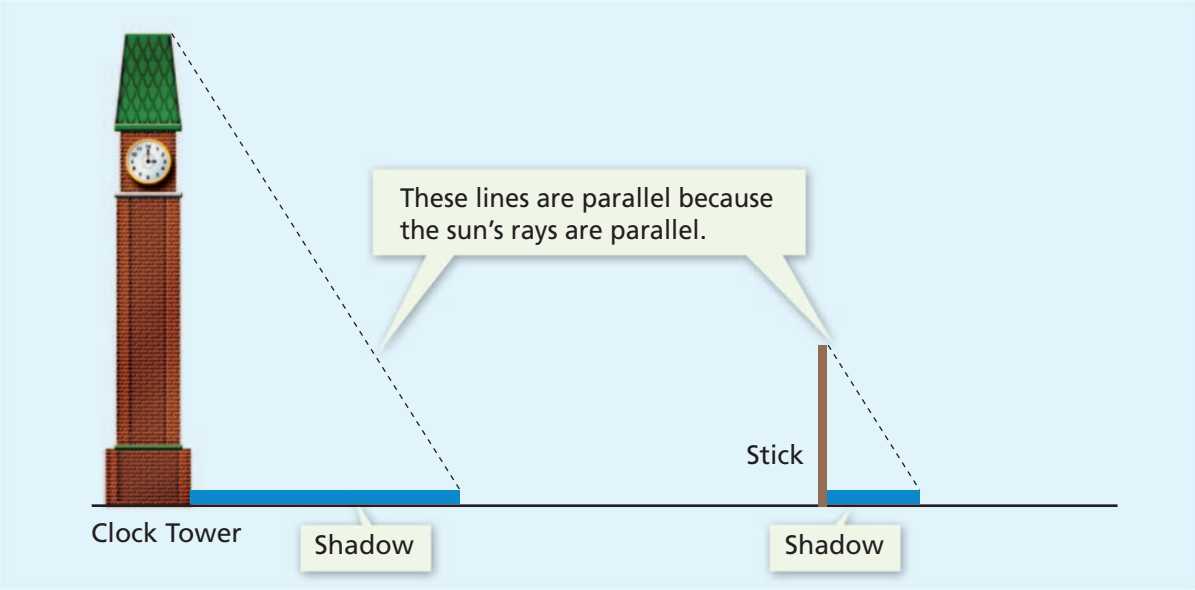
Investigation 5

Using Similar Triangles and Rectangles

You can find the height of a school building by climbing a ladder and using a long tape measure. You can also use easier and less dangerous ways to find the height. In this investigation, you can use similar triangles to estimate heights and distances that are difficult to measure directly.

5.1 Using Shadows to Find Heights

If an object is outdoors, you can use shadows to estimate its height. The diagram below shows how the method works. On a sunny day, any upright object casts a shadow. The diagram below shows two triangles.



A triangle is formed by a clock tower, its shadow, and an imaginary line from the top of the tower to the end of the shadow.

A triangle is formed by a stick, its shadow, and an imaginary line from the top of the stick to the end of its shadow.

Getting Ready for Problem 5.1

Examine the diagram of the shadow method. Why does each angle of the large triangle have the same measure as the corresponding angle of the small triangle? What does this suggest about the similarity of the triangles?

To find the height of the building, you can measure the lengths of the stick and the two shadows and use similar triangles.

Problem 5.1 Using Shadows to Find Heights

Suppose you want to use the shadow method to estimate the height of a building. You make the following measurements:

- length of the stick: 3 m
- length of the stick's shadow: 1.5 m
- length of the building's shadow: 8 m

A. Make a sketch of the building, the stick, and the shadows. Label each given measurement. What evidence suggests that the two triangles formed are similar?

B. Use similar triangles to find the building's height from the given measurements.

C. A tree casts a 25-foot shadow. At the same time, a 6-foot stick casts a shadow 4.5 feet long. How tall is the tree?

D. A radio tower casts a 120-foot shadow. At the same time, a 12-foot-high basketball backboard (with pole) casts a shadow 18 feet long. How high is the radio tower?

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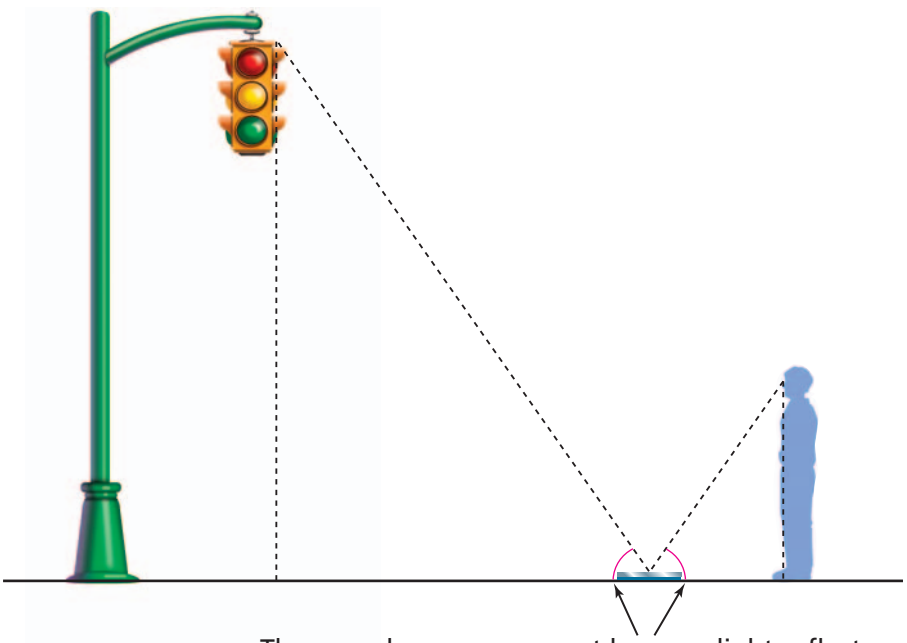
5.2 Using Mirrors to Find Heights

The shadow method only works outdoors on sunny days. As an alternative, you can also use a mirror to estimate heights. The mirror method works both indoors and outdoors.

The mirror method is shown below. Place a mirror on a level spot at a convenient distance from the object. Back up from the mirror until you can see the top of the object in the center of the mirror.

The two triangles in the diagram are similar. To find the object's height, you need to measure three distances and use similar triangles.



These angles are congruent because light reflects off a mirror at the same angle it arrives.

Getting Ready for Problem 5.2

Examine the diagram above. Explain why each angle of the large triangle has the same measure as the corresponding angle of the small triangle. What does this suggest about the similarity of the triangles?

Problem 5.2 Using Mirrors to Find Heights

A. Jim and Su use the mirror method to estimate the height of a traffic signal near their school. They make the following measurements:

- height from the ground to Jim's eyes: 150 cm
- distance from the middle of the mirror to Jim's feet: 100 cm
- distance from the middle of the mirror to a point directly under the traffic signal: 450 cm

1. Make a sketch. Show the similar triangles formed and label the given measurements.
2. Use similar triangles to find the height of the traffic signal.

B. Jim and Su also use the mirror method to estimate the height of the gymnasium in their school. They make the following measurements:

- height from the ground to Su's eyes: 130 cm
- distance from the middle of the mirror to Su's feet: 100 cm
- distance from the middle of the mirror to the gym wall: 9.5 m

1. Make a sketch. Show the similar triangles formed and label the given measurements.
2. Use similar triangles to find the height of the gymnasium.

C. Use the mirror method to find the height of your classroom. Make a sketch showing the distances you measured. Explain how you used the measurements to find the height of the room.

D. Compare the two methods (shadow or mirror) for finding missing measurements. What types of problems may arise when using these methods?

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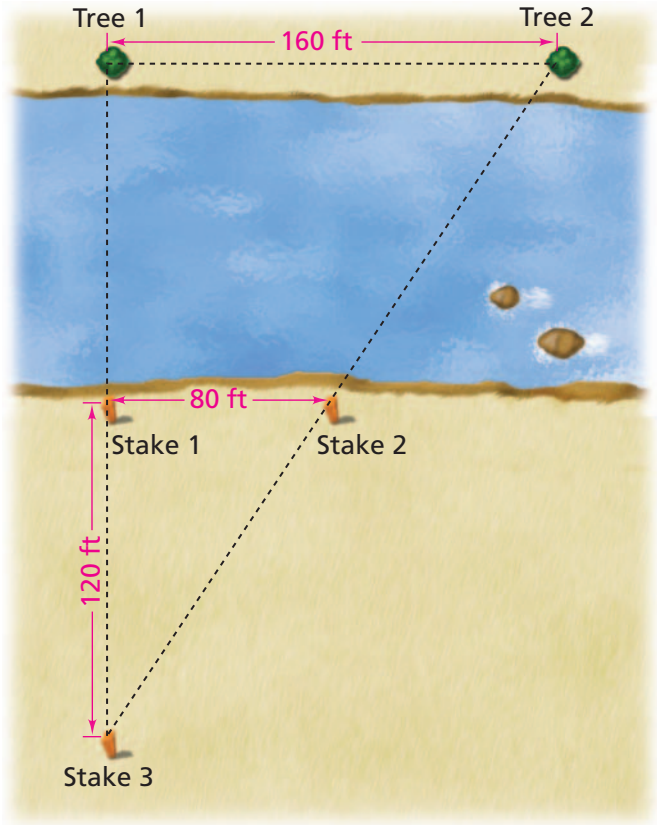
5.3 On the Ground . . . but Still Out of Reach

Darnell, Angie, and Trevor are at a park along the Red Cedar River with their class. They decide to use similar triangles to find the distance across the river. After making several measurements, they sketch the diagram below.



Getting Ready for Problem 5.3

In the two previous problems, you used the fact that if two triangles have corresponding angles with the same measure, then the triangles are similar. This is not true for other polygons in general.

- What do you know about parallelograms and rectangles that explains this?
- Which triangles in the river diagram are similar? Why?

Problem 5.3 Finding Lengths With Similar Triangles

A. Use the river diagram. Which triangles appear to be similar? Explain.

B. What is the distance across the river from Stake 1 to Tree 1? Explain.

C. The diagram shows three stakes and two trees. In what order do you think Darnell, Angie, and Trevor located the key points and measured the segments?

D. Another group of students repeats the measurement. They put their stakes in different places. The distance from Stake 1 to Stake 2 is 32 feet. The distance from Stake 1 to Stake 3 is 30 feet. Does this second group get the same measurement for the width of the river? Explain.

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